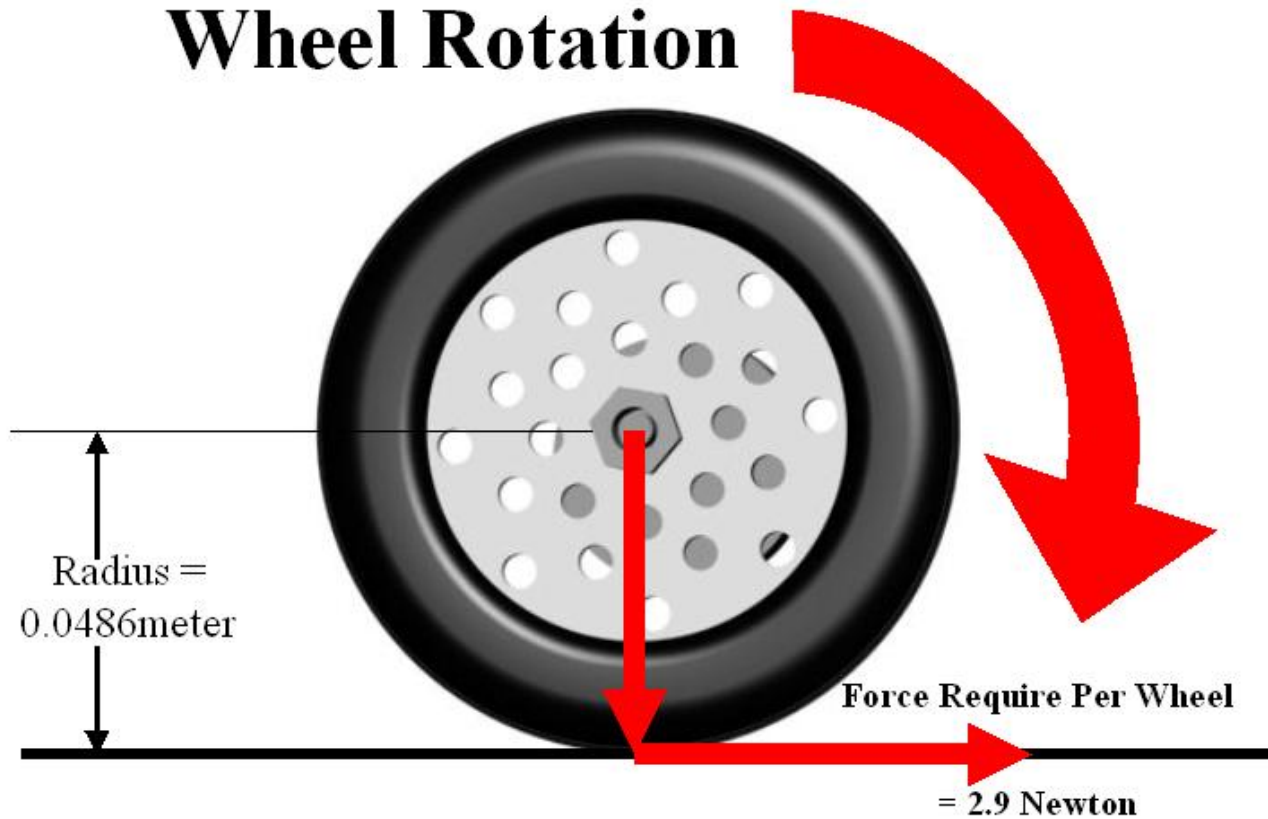


## Lesson 3

# Understanding and Using DC Motor Specifications

## Wheel Rotation



$$\text{Torque}_{\omega} = \text{Force} \times \text{Distance}$$

**Grade Level** 8-12 through Post Secondary  
**Estimated Time** 1-4 Hours

## Abstract

Engineering design success depends in great part on reducing the time spent creating modules, mechanisms and machines. The use of accurate mathematical models can speed up the design process, and minimize the time wasted on trial and error design methods.

Trial and error design methods are inefficient and costly. Matching a motor to a specific application is not easily accomplished through trial and error. Moreover, the necessity of purchasing and testing many dozens of motors is economically wasteful and time consuming.

Instead, the best engineering practices employ mathematical models early in the design process. The functional requirements and design parameters of a motor system can be determined early in the design process and manufacturer's published motor data can be researched in an attempt to find a suitable match.

Designers and engineers use mathematical models to optimize the time spent designing modules, mechanisms and machines by:

- Reducing the number of possible items in the selection set.
- Reducing testing and prototyping time
- Providing a closer match of motor performance to the functional requirements and design parameters specified early in the design process.

This lesson serves to demonstrate how an understanding of motor performance specifications and mathematical models can be used to more closely match the performance of a DC motor system to a pre-determined specification. This lesson makes extensive use of the experimental data obtained in the previous lesson. In addition, students who participate in this lesson will gain experience in researching manufacturer's published motor data in an effort to best match a DC motor system to an existing specification.

## Objectives

Students who participate in this lesson, and the related activities will:

- Use experimental data to create mathematical models of motor performance
- Use experimentally determined motor performance data along with manufacturer's published data to make useful predictions about DC motor performance.
- Use their understanding of Newton's laws of motion, torque and rotational speed to compute the performance requirements of a DC motor system.
- Use engineering methods and mathematical models in an effort to most closely match DC motor performance to a given set of functional requirements and design parameters.

## Terms and Concepts

Stall Current	Voltage	Multimeter
Torque Constant	Back emf Constant	No Load RPM
Electromotive Force	Stall Torque	Mechanical Advantage
Dynamometer	Mass	Gear Ratio
Resistance	Acceleration	Torque
Radians per Second	Ohm's Law	

## Materials/Equipment/Supplies/Software

GEARS-IDS™ Kit	Calculator	Heavy Duty 12V Battery
Internet Access	Multimeter (2)	

**Note: Before beginning this lesson, it will be necessary to review and understand how to use Ohms Law (  $E = I \cdot R$  ) to calculate voltage (E) when both the current (I) and resistance (R) is given.**

## Using Experimental or Published Data to Create Mathematical Models of Motor Performance

Let's look at the data obtained through experiments with the gear head motors supplied in the GEARS-IDS™ kit. Listed below are the experimental results and the manufacturers published specifications:

**Stall Current** = 12.06 Amps @ 15.1 volts (Manufacturers Specs) = 0.79 Amps/Volt

**Stall Current** = 9.7 Amperes at 12.5 volts (Experimental) = 0.76 Amps/Volt

**K<sub>E</sub> (Voltage constant)** = 2.43 volts/Thousand RPM (Manufacturers Specs) = 0.0232 volts/rad/sec.

**K<sub>E</sub> (Voltage constant)** = 2.43 volts/Thousand RPM (Experimental) = 0.0232 volts/rad/sec.

**K<sub>T</sub> (Torque Constant)** = 3.21 oz-in./ampere (Manufacturers Specs)

**K<sub>T</sub> (Torque constant)** = 3.02 oz-in../ampere (Experimental)

### Model GM9X34 Winding Data (Other windings available upon request)

Line No.	Parameter	Symbol	Units	9X34			
58	Reference Voltage	E	V	12.0	19.1	24.0	30.3
59	Torque Constant	K <sub>T</sub>	oz.in./A	(18.2 X 10 <sup>-3</sup> )	(28.7 X 10 <sup>-3</sup> )	(36.5 X 10 <sup>-3</sup> )	(45.9 X 10 <sup>-3</sup> )
60	Back-EMF Constant	K <sub>E</sub>	V/krpm (V/rad/s)	1.91 (18.2 X 10 <sup>-3</sup> )	3.01 (28.7 X 10 <sup>-3</sup> )	3.82 (36.5 X 10 <sup>-3</sup> )	4.81 (45.9 X 10 <sup>-3</sup> )
61	Resistance	R <sub>T</sub>	Ω	0.83	1.89	2.96	4.62
62	Inductance	L	mH	0.63	1.56	2.51	3.97
63	No-Load Current	I <sub>NL</sub>	A	0.33	0.21	0.16	0.13
64	Peak Current (Stall)	I <sub>p</sub>	A	14.5	10.1	8.11	6.55

Note: The Torque Constant K<sub>T</sub> shown in the table above is called out as Oz.in./Ampere but is given instead as Newton-meters/ampere. This is a manufacturer's publishing error that was not addressed at the time of this printing. To convert N-m/A to oz.in./A simply multiply N-m/A x 141.611.

## Use the data from the previous page to make some useful predictions about the DC gear head motor found in the GEARS-IDS™ Kit.

To do this we must look at a couple of useful algebraic expressions:

$$T = K_T \cdot I$$

$$E = K_E \cdot \omega$$

Where

**K<sub>T</sub>** – Torque Constant is given in N-m/A (Newton-meters/Ampere) or oz.-in/A (Oz.-in/Ampere)

**K<sub>E</sub>** - Voltage constant is given in volts/radians/seconds or volts/krpm (Volts per Thousand rpm)

**T** - Torque, is given in Nm (Newton-meters)

**I** - Current is given in Amps (Amperes)

**E** - EMF, is given in Volts

**ω** - Angular velocity, is given radians/sec and/or revolutions per second

Remember, when K<sub>T</sub> (Torque constant) and K<sub>E</sub> (Voltage constant) are given in Amps/Nm and Volts/Radians/sec respectively, they have the same numerical values.

Thus

$$\begin{aligned} \mathbf{K_E} \text{ (Voltage constant)} &= \mathbf{K_T} \text{ (Torque constant)} \\ 0.0232 \text{ volts/rad/sec.} &= 0.0232 \text{ Nm/Ampere} \end{aligned}$$

Given this information, and the two equations listed above, it is possible to predict the torque of the motor at any current draw up to stall current. In addition, we can determine the rotational speed (Angular velocity) of the motor armature at any given voltage.

### Using the K<sub>E</sub> (Voltage constant)

Suppose you wished to determine the shaft RPM of the Pittman gear head motors used in the GEARS-IDS™ Kit, at an applied battery voltage of 12.6 volts?

Begin by considering the operation of the motor.

When a fixed magnet DC motor (With no load acting on it) is connected to a battery the armature will rotate at a speed and direction dependent on the battery voltage and



polarity. Theoretically the motor armature should continue to accelerate to a higher and higher speed unless there is a force that works in opposition to the battery voltage. Clearly the motor armature does not continue to accelerate but instead reaches a finite top speed.

The motor shaft will not continue to rotate faster because the spinning of the armature coils within the permanent magnetic field of the motor generates a voltage or a back emf that opposes the applied voltage.

Under these circumstances there are several forces at work.

1. The forward emf (Voltage) supplied by the battery
2. The back emf (Voltage) generated by the armature spinning within the fixed magnetic field of the motor.

*Note: The relationship between the back emf (voltage) and the rotational speed of the armature is given as the  $K_E$  (Voltage constant).*

3. The emf or voltage drop ( $I \times R$ ) across the internal resistance of the armature. The resistance of the armature is relatively slight in comparison to the back emf effect.

In order for the motor armature to continue to spin, a current must continue to pass through the armature windings. The battery voltage must therefore supply enough emf (voltage) to equal both the back emf (voltage) induced within the armature windings as well as the emf (voltage drop) across the armature resistance.

By setting the supplied voltage equal to the back emf (the  $K_E$ ) at a chosen (fixed) rotational speed and adding the voltage drop across the armature resistance ( $I \times R$ ), we can create an algebraic expression that will allow us to calculate the rotational speed of the armature when any given voltage is applied to it.

Using the voltage constant,  $K_E$ , we find that the voltage induced in the motor armature can be calculated for any rotational speed by multiplying the  $K_E$  (Voltage Constant) x the rotational speed of the armature. **Note: The voltage constant,  $K_E$  can be determined experimentally as described in the previous chapter, or obtained from the manufacturer's published specifications.**

$$E_{induced} = K_E \cdot \omega$$

The voltage drop across the armature resistance can be calculated using Ohm's law. In order to calculate the voltage drop however we will need to determine the "No load" current through the armature as it turns against only the internal resistance of the bearings and gearbox. The current is measured while the motor is free running at the specified voltage of 12.6 volts. This current is measured at 0.238 Amperes.

$$E_{drop} = I \cdot R_{armature}$$
$$E_{drop} = 0.238 \text{ Amperes} \cdot 1.26 \Omega = 0.30 \text{ Volts}$$

The battery voltage is a variable that can be determined by the investigator. In this case we elect to calculate the rotational speed of the motor armature at 12.6 volts.

$$E_{battery} = 12.60 \text{ Volts}$$

We can now write an expression that sets the battery voltage  $E_{Battery}$  equal to the resistance losses in the armature, plus the counter emf induced in the winding.

$$E_{battery} = K_E \cdot \omega + I \cdot R_{armature}$$

$$12.60V = 0.0232v / rad / sec \cdot \omega + 0.238Amperes \cdot 1.26\Omega$$

Rewriting the equation in terms of the rotational speed or angular velocity of the armature yields the following expression, which can be used to calculate the rotational speed of the motor for any applied voltage.

$$\omega = \frac{(E_{battery} - I \cdot R)}{K_E}$$

$$\omega = \frac{(12.60V - 0.238Amperes \cdot 1.26\Omega)}{0.0232v / rad / sec}$$

$$\omega = 530.2Radians / sec$$

or

$$\omega = 5063.9RPM$$

Note: Convert Radian/sec and RPM using the following formulae;

$$RPM = \frac{Radians / sec}{\frac{2\pi}{60}} \quad \text{Conversely} \quad Radians / sec = \frac{2\pi}{60} \cdot RPM$$

Now we will consider the transmission gearing. The transmission (reduction) ratio is 19.7:1. Calculate the shaft speed by dividing the armature speed by the transmission ratio. This will yield the expected rotational speed of the Pittman gear head motor output shaft.

$$Output \text{ Shaft Speed} = \frac{5063.9RPM}{19.7} = 257RPM$$

Actual measurements of the output shaft (*free running*) speed at 12.6 volts yield a measured 256 RPM! It's nice to know the math works!

## Using the $K_T$ (Torque constant)

Suppose you wished to determine the motor torque at  $\frac{1}{2}$  the stall current.

Using the torque constant equation above, we find the following relationship between the expected motor torque  $T$ , the Torque constant,  $K_T$  and the current  $I$ .



$$T = K_T \cdot I \quad \text{dividing the stall current by 2 yields} \quad T = K_T \cdot \frac{I}{2}$$

Since  $K_T$  and  $K_E$  have the same values when the following units are applied:

Torque,  $T$ , is given in Nm (Newton meters)

Current,  $I$ , is given in Amps (Amperes)

EMF,  $E$ , is given in Volts and

Angular velocity  $\omega$ , or rotational speed, is given radians/sec

We can substitute the (experimental) value of  $K_E$  for  $K_T$  in the equation above. The resulting answer will give us the expected torque output in Nm (Newton meters)

$$T = 0.0232 \text{ Nm / Ampere} \cdot \frac{\text{StallCurrent}}{2}$$

$$T = 0.0232 \text{ Nm / Ampere} \cdot \frac{9.5 \text{ Amperes}}{2}$$

$$T = 0.1102 \text{ Nm}$$

## Choosing the Correct DC Motor for a Specific Application

Design engineers are often faced with having to determine the best DC motor choice for a given functional requirement or design parameter. A typical example might look like this:

**Functional Requirement:** DC Gear head motor capable of accelerating a 15lb, two-wheel drive robot with wheel diameters of 3.825" at a rate of 3ft/sec/sec. Top speed required will be around 4 feet/sec

**Design Parameters:** Supplied Voltage = 12Volts, Motor size limited to an overall diameter of approximately 2" and an overall length of not more than 4" (*Less the output shaft length.*)

Design engineers are not always in a position to be assured their decision is the best given the universe of possible gear head motor options. Their decisions are often constrained by one of more of these 5 concerns:

### 1.Time 2.Money 3.Knowledge 4.Power and 5.Weight

In order to best solve this design problem, an engineer might choose to accomplish the following tasks in this order:

1. Calculate the required wheel torque and RPM described in the functional requirement
2. Research DC motor manufacturers to determine the availability of gear-head motors that meet the Functional Requirements and the Design Parameters.
3. Compare the published specifications of the various gear-head motors to determine which will provide the best value in terms of the constraints listed above.
4. Choose one or several motors for prototyping and testing.

## Step One: Calculate the Required Wheel Torque and RPM

**Review the functional requirement:** DC Gear head motor capable of accelerating a 15lb, two-wheel drive robot with wheel diameters of 3.825" at a rate of 3ft/sec/sec.

**Determine the torque (force) that must be generated at each wheel in order to meet the functional requirement.**

Convert all units to SI units (Use <http://www.onlineconversion.com> )

	Imperial	SI
Mass	= 15lbs	6.804 kg
Acceleration	= 3ft/square second	0.9144 meter/square second
Torque	= ft/lbs	Newton/meters

**Calculate required force in SI units**

$$F = MA$$

$$Force_{Total} = Mass_{kg} \cdot Acceleration_{Meters / second^2}$$

$$Force_{Total} = 6.804kg \cdot 0.99144m / s^2$$

$$Force_{Total} = 5.8_{Newtons}$$

The Total force required to meet the Functional Requirement is 5.8 Newtons. However, the vehicle has 2 motors and wheels. Therefore each motor/wheel combination needs only supply half the required force or 2.9 Newtons.

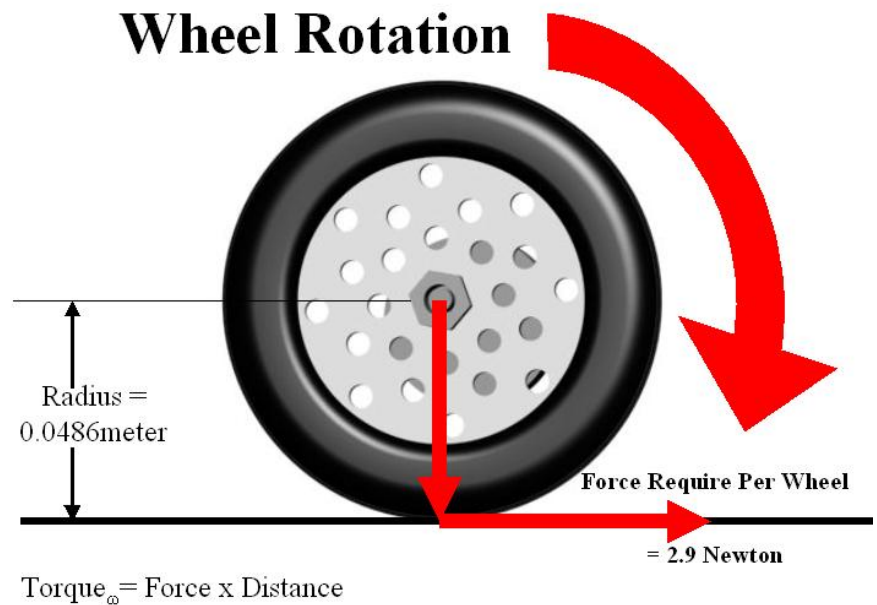
$$Force_{Wheel} = 2.9_{Newtons}$$



**Calculate the torque requirement for each wheel.**

*Use SI units*

Torque is a twisting or turning force that acts on an axis or pivot. The 2 wheels of this vehicle turn on an axis or shaft. The distance from the center of the turning axis to the outside of the wheel is the radius of the wheel. The torque or turning force of the axle acts through the radial distance as illustrated in the graphic below.



$$Torque = Force \cdot Distance$$

Distance	= Wheel Radius	= Wheel Diameter/2	= 3.825"/2 inches	= 0.0486meter
Force	= Force required per wheel		= 2.9 Newtons	

Remember: Keep all the units in the SAME system!

$$Torque_{wheel} = 2.9_{Newtons} * 0.0486_{meters}$$

$$Torque_{wheel} = 0.14094 \text{ Newton meters}$$

or it can be expressed in milli Newton meters by multiplying by 1000. Many manufacturer's of fractional horsepower motors express torque in milli Newton meters.

$$Torque_{wheel} = 140.94 \text{ milli Newton meters or mNm}$$

The required torque at each wheel is 0.14094 Newton meters.

**Note: Remember that DC motors develop less torque as the armature rotational speed increases. For this reason we will need to find a motor that can produce 0.14094 Newton meters of continuous torque at or near the design rpm of 250 revolutions per minute.**

## Calculate the required wheel RPM to maintain a speed of 4 feet per second

$$\begin{aligned}\text{Wheel Diameter} &= 3.825'' &= & 0.0971 \text{ meters} \\ \text{Wheel Circumference} &= \pi \times \text{Diameter} &= & 3.142 \times 0.0971 \text{ meters} = \mathbf{0.305 \text{ meters}} \\ \text{Required Speed} &= 4\text{ft/sec} &= & \mathbf{1.219 \text{ meters/second}} \\ \text{RPS Revolutions Per Second} & &= & \text{rpm}/60\end{aligned}$$

**Remember: Keep all the units in the SAME system!**

$$\text{Speed}_{\text{meters/second}} = \text{RPS} \cdot \text{Circumference}_{\text{meters}}$$

$$1.219_{\text{meters/second}} = \text{RPS} \cdot 0.305_{\text{meters}}$$

$$\text{RPS} = \frac{1.219_{\text{meters/second}}}{0.305_{\text{meters}}}$$

$$\text{RPS} = 3.996 \quad \text{RPM} = 239.76$$

## Step Two: Research manufacturers in order to determine the availability of gear-head motors that meet the functional requirements and the design parameters.

List the required torque and Wheel velocity from the previous calculations

$$\begin{aligned}\text{Required torque} &= 0.1409 \text{ Newton meters} &= & 1.247 \text{ inch pounds} &= & 19.95 \text{ inch ounces} \\ \text{Wheel velocity} &= 239 \text{ Revolutions per minute} &= & 3.996 \text{ Revolutions per second} &= & 25 \text{ Radians sec}\end{aligned}$$

Note: It is important to calculate the motor specifications using one set of units, but it is also necessary to list the specifications in a variety of units since individual manufacturers use different units.

For the purpose of choosing a motor for this lesson activity, we will consult the Pittman Motor Company specifications.

### Review the functional requirements and the design parameters:

**Functional Requirement:** DC Gear head motor capable of accelerating a 15lb, two-wheel drive robot with wheel diameters of 3.825'' at a rate of 3ft/sec<sup>2</sup>. Top speed required will be around 4 feet/sec

**Design Parameters:** Supplied Voltage = 12Volts, Motor size limited to an overall diameter of approximately 2'' and an overall length of not more than 4'' (*Less the output shaft length.*)

*Note: Motor manufacturers publish their motor specifications online as a convenience to engineers and designers looking to make motor purchases. For the most part these motor manufacturers are expecting to sell caseloads of motors to OEM's (Original Equipment Manufacturers) and are not expecting to sell individual motors.*

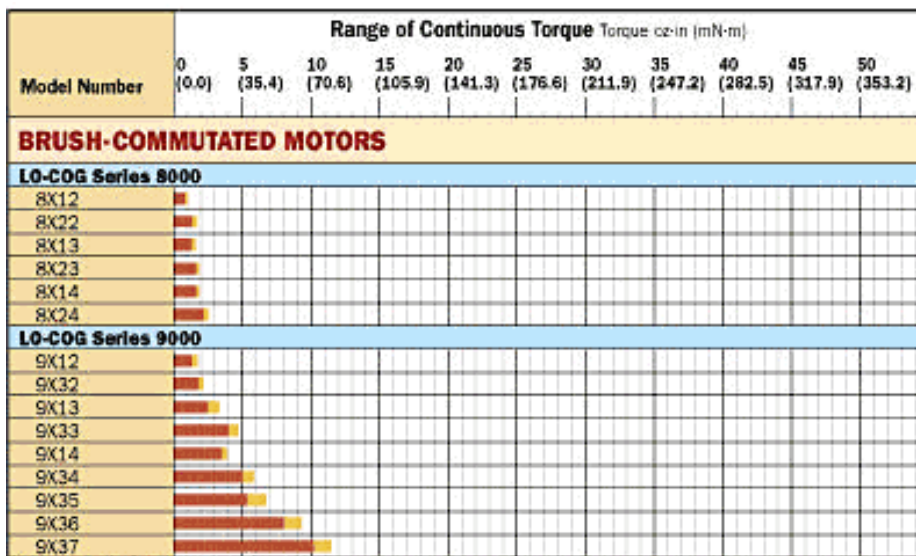
Pittman Motors/Penn Engineering website: [http://www.pennmotion.com/quick\\_index.html](http://www.pennmotion.com/quick_index.html)

- 1.) Visit this web link.
- 2.) Click on the "Torque Selector Charts" link near the top of the page <http://www.pennmotion.com/continoustorqueselector.html>
  - a. A quick review of the chart shows torque outputs for 3 "Series" of brush commutated motors. These are 8000 series, 9000 series and 14000 series. We need a motor with a continuous torque output of at least 140.9 mNm or 20 oz. in. of torque.
  - b. Reviewing the product series to find the motor torques that best fit the calculated requirements suggests the use of a 14000 series motor. This will not be the best decision, as we will see in the next step.
- 3.) Return to the **Penn Engineering website:** [http://www.pennmotion.com/quick\\_index.html](http://www.pennmotion.com/quick_index.html)
  - a. Select the 8000-9000-14000 series motor link by clicking on the photo of the brush motors. Click on the Tech Specs link. (This would be bulletin LCM which stands for Low Cog Motors) [http://www.pennmotion.com/pdf/lcm\\_bulletin.pdf](http://www.pennmotion.com/pdf/lcm_bulletin.pdf)
  - b. Scroll down to the 14000 series Motor Data on page 14. Read across Line 1 Continuous Torque until you reach the 14XX3 Column. The torque value listed matches the motor specifications that we described. However, The No-Load Speed listed on line 4 is 3456 rpm! This is way more motor than we require since we are looking for a motor that will yield 20 oz.in of torque at only 250 rpm.

### Motor Data

Line No.	Parameter	Symbol	Units	14XX1	14XX2	14XX3	14XX4	14XX5	14XX6	14XX7
1	Continuous Torque (Max.) <sup>1</sup>	T <sub>C</sub>	oz-in (N-m)	10.0 (70.6 X 10 <sup>-3</sup> )	14.0 (98.9 X 10 <sup>-3</sup> )	21.0 (148.3 X 10 <sup>-3</sup> )	26.0 (183.6 X 10 <sup>-3</sup> )	31.0 (218.9 X 10 <sup>-3</sup> )	36.5 (257.8 X 10 <sup>-3</sup> )	50.0 (353.1 X 10 <sup>-3</sup> )
2	Peak Torque (Stall) <sup>2</sup>	T <sub>PK</sub>	oz-in (N-m)	62.8 (.44)	107 (.76)	159 (1.12)	204 (1.44)	225 (1.59)	284 (2.01)	410 (2.90)
3	Motor Constant	K <sub>M</sub>	oz-in/√W (N-m/√W)	4.45 (31.4 X 10 <sup>-3</sup> )	5.93 (41.9 X 10 <sup>-3</sup> )	7.88 (55.6 X 10 <sup>-3</sup> )	8.63 (60.9 X 10 <sup>-3</sup> )	9.97 (70.4 X 10 <sup>-3</sup> )	10.9 (77.0 X 10 <sup>-3</sup> )	13.1 (92.5 X 10 <sup>-3</sup> )
4	No-Load Speed	S <sub>NL</sub>	rpm (rad/s)	4230 (443)	4087 (428)	3456 (362)	3702 (388)	3056 (320)	3216 (337)	3211 (336)


Since the rpm we require is so low, we can choose a motor series with less torque and use a transmission to lower the rpm while raising the torque. We will now return to the Pittman website and look at **Gear Head Motors.**




- 4.) Return to the **Penn Engineering website:** [http://www.pennmotion.com/quick\\_index.html](http://www.pennmotion.com/quick_index.html) and click on the "Torque Selector Charts" link near the top of the page. <http://www.pennmotion.com/continoustorqueselector.html>

This time we will choose a motor that has a least 25% of the specified torque or around 5 oz-in. of torque. The 9x34 series motor is one of several choices we can make. Let's start with that selection. This time however we will select from the Gear Head motors on the Pittman website.

5.) Return to the **Penn Engineering website:** [http://www.pennmotion.com/quick\\_index.html](http://www.pennmotion.com/quick_index.html) and scroll down to the GM8000 - GM9000 – GM14000 series motors. The prefix GM stands for Gear Motor. The correct selection is shown on the preceding page. Scroll down to the GM9000 series Motor Data on page 9 of 20, and find the continuous torque rating for the 9x34 series Gear Motor.

Click the photo for more description		TOP	
Bulletin LCM			
		SERIES	GENERAL INFORMATION
		8000 9000 14000	Pittman LO-COG® brush-commutated servo motors feature 7- or 11-slot armature designs and are available in 3 models with 4 standard windings each.  <a href="#">PITTMAN</a>

Click the photo for more description		TOP	
Bulletin LCG			
		SERIES	GENERAL INFORMATION
		GM8000 GM9000 GM14900	Pittman LO-COG® gearmotors feature spur gearheads with a variety of options and provide output torques up to 175 oz.in with standard gearing, 500 oz.in with optional gearing  <a href="#">PITTMAN</a>

The continuous torque rating for the GM9X34 series Gear Motor is listed as 6.1 oz-in. We will need to select an appropriate gear reduction in order to increase the torque and match the rpm specification. Since it is unlikely that we will exactly match both the torque and the rpm specified, we will choose to more closely match the rpm and accept the resulting torque.

### Motor Data

Line No.	Parameter	Symbol	Units	GM9X32	GM9X33	GM9X34	GM9X35	GM9X36
30	Continuous Torque (Max.) <sup>3</sup>	T <sub>C</sub>	oz-in (N-m)	2.3 (16.2 X 10 <sup>-3</sup> )	4.7 (33.2 X 10 <sup>-3</sup> )	6.1 (43.1 X 10 <sup>-3</sup> )	6.9 (48.7 X 10 <sup>-3</sup> )	9.5 (67.1 X 10 <sup>-3</sup> )
31	Peak Torque (Stall) <sup>4</sup>	T <sub>PK</sub>	oz-in (N-m)	13.8 (97.5 X 10 <sup>-3</sup> )	31.6 (223.2 X 10 <sup>-3</sup> )	41.3 (291.7 X 10 <sup>-3</sup> )	49.4 (348.9 X 10 <sup>-3</sup> )	61.8 (436.4 X 10 <sup>-3</sup> )
32	Motor Constant	K <sub>M</sub>	oz-in/√W (N-m/√W)	1.62 (11.4 X 10 <sup>-3</sup> )	2.66 (18.8 X 10 <sup>-3</sup> )	3.01 (21.3 X 10 <sup>-3</sup> )	3.21 (22.7 X 10 <sup>-3</sup> )	4.11 (29.0 X 10 <sup>-3</sup> )

6.) Scroll up the Pittman document [http://www.pennmotion.com/pdf/lcg\\_bulletin.pdf](http://www.pennmotion.com/pdf/lcg_bulletin.pdf) to page 8 of 20 where you will find the no load RPM data for the gearmotor. Note the No-Load speed for

Gearmotor Data			Reduction Ratios												
Line No.	Parameter	Symbol	Units	5.9:1	11.5:1	19.7:1	38.3:1	65.5:1	127.8:1	218.4:1	425.9:1	728.1:1	1419.8:1	2426.9:1	4732.5:1
<b>NO-LOAD SPEED (All Gears)</b>															
22	GM9X32	S <sub>NL</sub>	rpm (rad/s)	1189 (125)	610 (63.9)	357 (37.4)	183 (19.2)	107 (11.2)	54.9 (5.75)	32.1 (3.36)	16.5 (1.73)	9.6 (.513)	4.9 (.513)	2.9 (.304)	1.5 (.157)
23	GM9X33	S <sub>NL</sub>	rpm (rad/s)	1016 (106)	521 (54.6)	305 (31.9)	156 (16.3)	91.5 (9.58)	46.9 (4.91)	27.4 (2.87)	14.1 (1.48)	8.2 (.859)	4.2 (.440)	2.5 (.262)	1.3 (.136)
24	GM9X34	S <sub>NL</sub>	rpm (rad/s)	1043 (109)	535 (56.0)	313 (32.8)	160 (16.8)	93.9 (9.83)	48.1 (5.04)	28.2 (2.95)	14.4 (1.51)	8.5 (.890)	4.3 (.450)	2.5 (.262)	1.3 (.136)

the 9X34 Gear motor with a 19.7 to 1 gear reduction. This speed is listed as 313 RPM. This is close to ideal since in our application there WILL be a load on the gear motor.

7.) Let's calculate the torque available at the transmission output shaft of the 9X34 Gear Motor with a 19.7:1 gear reduction transmission.

We will need to look at the input torque from the motor, the mechanical advantage (Torque Multiplier) gained through the transmission gearing and the mechanical advantage lost through the rotational mass and friction of the transmission gears (*Transmission efficiency*).

We will need to record some of the manufacturer's data that we have already reviewed.

9X34 <b>Continuous</b> (Motor) Torque	6.1 oz-in. (Remember, This is the torque the motor puts into the transmission)
9X34 Gear Reduction	19.7:1
9X34 Transmission Efficiency	73% (The sum of all the inertia and friction losses through the transmission gears and shafts)

The expression we will use looks like this:

**Out Put Torque (Tran. shaft) = Input Torque (Motor) x Gear Reduction x Transmission Efficiency**

$$\text{Output Torque} = 6.1 \text{ oz-in.} \times 19.7 \times .73$$

$$\text{Output Torque} = 87.7 \text{ oz-in. of Continuous Torque}$$

The 9X34 series Gear Head motor is more than adequate for the requirements we specified. The rpm is slightly more than we specified and the torque is more than 4 times what we require. The overall dimensions of this component almost exactly matches our specifications. This motor leaves us with some design "Head Room". It is not a perfect match, but it is fairly close. The next important consideration would be cost. If there is an 8000 series gear head motor that more closely matches our requirements, then it would be our responsibility as design engineers to make that determination.

The challenge offered to you is this:

Using the published Pittman data, find an 8000 series motor that most nearly matches the functional requirements and design parameters listed above. Then call Pittman Express and obtain a price quote for the motor you determined most closely meets the requirements we described. You can repeat this process for additional motor manufacturers.

Good Luck, your on your way to becoming DC motor experts.!

## Lesson 3 Work Sheet

# Using Experimental and Published Data to Create Mathematical Models of Motor System Performance

Name \_\_\_\_\_ Date \_\_\_\_\_ Class \_\_\_\_\_

**Directions: Answer the following questions using the information provided in this lesson.**

- 1.) \_\_\_\_\_ Stall Current is a measure of the \_\_\_\_\_ amount of current draw through the armature winding.  
A.) Maximum B.) Minimum C.) Average D.) Continuous
- 2.) \_\_\_\_\_ Stall current is measured in \_\_\_\_\_.  
A.) Volts B.) Amperes C.) Ohms D.) Watts
- 3.) \_\_\_\_\_ Current is proportional to this force in a brush commutated DC motor.  
A.) Resistance B.) Acceleration C.) RPM D.) Torque
- 4.) \_\_\_\_\_ The Torque Constant is measured in the following units.  
A.) Volts/rad/sec B.) Amperes/rpm C.) Amperes/oz-in. D.) Oz-in./Ampere
- 5.) \_\_\_\_\_ The symbol for the Torque Constant is \_\_\_\_\_.  
A.)  $\Omega$  B.)  $K_T$  C.)  $K_E$  D.) I
- 6.) \_\_\_\_\_ A motor generates 15 oz-in. of torque at 3 amperes. The torque constant is \_\_\_\_\_.  
A.) 5 amperes/oz-in. B.) 3 oz-in./ampere C.) 5 oz-in./ampere D.) 3 amperes/oz-in.
- 7.) \_\_\_\_\_ The DC motor voltage constant is measured in the following units.  
A.) Rpm/thousand volts B.) Volts/Thousand rpm C.) Amperes D.) Volts
- 8.) \_\_\_\_\_ Moving charges or current are measured in which of the following units?  
A.) Amperes B.) Volts C.) Watts D.) Both A and C
- 9.) \_\_\_\_\_ The symbol for the voltage constant is \_\_\_\_\_.  
A.)  $\Omega$  B.)  $K_T$  C.)  $K_E$  D.) I
- 10.) \_\_\_\_\_ Increasing the voltage supplied to a brush commutated DC motor causes a proportional increase in the motor \_\_\_\_\_.  
A.) Resistance B.) Torque C.) Rotational Speed D.) Current
- 11.) \_\_\_\_\_ A remotely operated vehicle with a mass of 5 kg accelerates at a rate of 0.5 meters sec/sec. What is the (minimum) net force acting to accelerate the vehicle?  
A.) 12.5 Newtons B.) 0.25 Newtons C.) 1.25 Newtons D.) 2.5 Newton
- 12.) \_\_\_\_\_ The voltage constant is also referred to as the \_\_\_\_\_.  
A.) Back-EMF constant B.) Speed constant C.) Power constant D.) Motor constant

- 13.) \_\_\_\_\_ Ohms law describes the relationship between Voltage, Current and Resistance and is expressed as \_\_\_\_\_.  
 A.)  $E = I * R$     B.)  $E = I/R$     C.)  $I = R/E$     D.)  $R = I * E$
- 14.) \_\_\_\_\_ The E in Ohms Law represents \_\_\_\_\_.  
 A.) Voltage    B.) Amperage    C.) Current    D.) Resistance
- 15.) \_\_\_\_\_ A DC motor armature draws 0.380 amperes while running at a no load speed of 5000 rpm. The manufacturer lists the armature resistance as being 3.25 ohms. What is the voltage drop across the armature windings?  
 A.) 1.235 Amperes    B.) 1.235 Volts    C.) 12.35 Amperes    D.) 12.35 Volts
- 16.) \_\_\_\_\_ A DC motor having a Torque Constant of 3.5 oz-in./ampere draws 5 amperes of current. Calculate the torque produced at the armature shaft.  
 A.) 17.5 Amperes    B.) 17.5 oz-in.    C.) 7.5 Amperes    D.) 7.5 oz-in.
- 17.) \_\_\_\_\_ Ohm's law is an algebraic expression that describes the relationship between Current, Voltage and \_\_\_\_\_ in a circuit.  
 A. Resistance    B.) Impedance    C.) Amperage    D.) The magnetic field
- 18.) \_\_\_\_\_ The no load speed of a motor is listed by the manufacturer as being 745 radians/sec what is the speed in rpm?  
 A.) 711.4 rpm    B.) 71.14 rpm    C.) 7114 rpm    D.) 71146 rpm
- 19.) \_\_\_\_\_ The back EMF constant can be used to evaluate a motors ability to generate electrical energy.  
 A.) True    B.) False
- 20.) \_\_\_\_\_ The no load speed of a motor is listed by the manufacturer as being 5000 rpm what is the speed in radians/second?  
 A.) 500    B.) 623    C.) 478    D.) 523
- 21.) \_\_\_\_\_ A robot wheel has a radius of 2.75 inches. How far will it travel in one complete revolution (*approximately*) ?  
 A.) 17"    B.) 15"    C.) 17.28"    D.) 15.3"
- 22.) \_\_\_\_\_ A motor has a voltage constant,  $K_E$  equal to 2.96 volts/thousand rpm. What voltage will be produced across the motor leads if the motor armature shaft spins at 4320 rpm?  
 A.) 12.78    B.) 12787.7    C.) 11.9    D.) 13.76
- 23.) \_\_\_\_\_ A motor supplied with 10 volts, draws 12.8 amperes of current at stall. What is the approximate armature resistance? Hint: Use Ohm's Law  $E = I * R$   
 A.) 7.81 ohms    B.) 78.1 ohms    C.) 7.81 ohms    D.) 0.781 ohms
- 24.) \_\_\_\_\_ The  $K_E$  (Voltage constant) of a given motor is 0.0445 volts/radians/sec. The torque constant of this same motor will be \_\_\_\_\_.  
 A.) 0.0232 Amperes/Nm    B.) 0.0445 Amperes/Nm    C.) 0.0232 oz-in/Ampere    D.) A and C
- 25.) \_\_\_\_\_ The Back emf generated in a spinning motor armature opposes current flow through the armature.  
 A.) True    B.) False



# Research Project Activity

## Reading and Interpreting Manufacturer's Motor Specifications

### Purpose

This activity is a very real exercise in how engineers obtain, interpret and use and share data. One of the more compelling reasons for acquiring technical literacy is to develop the ability to access, interpret and share data and information.

Engineers and technical workers are expected to acquire the knowledge and skills necessary to research, read and understand technical specifications. In addition, they are expected to recognize and convert measurements and units so they can make direct comparisons between products that are described using Imperial, SI or metric values.

This exercise challenges the students understanding of DC motor technology, as well as their fact finding and problem solving abilities. This exercise can seem like a daunting task, but by working in groups, organizing their data and communicating their findings, they will be able to succeed.

### Description

Using the internet, locate DC motor manufacturers who publish specifications for the small brush commutated DC motors they manufacture. Using the manufacturers published data create a list of motor choices that most nearly match the motor design parameters and functional requirements listed below. Present the two best choices you are able to find and explain why your group believes they have found the best options.

Present your findings using 1 slide for each motor and a spread sheet that compares the motor options your group was able to find.

A Partial list of manufacturers include:

Pittman Motors <http://www.pennmotion.com>  
Buelher Motor <http://www.buehlermotor.com/>  
Johnson Electric <http://www.johnsonelectric.com/product/product.php>  
Faulhaber Motors <http://www.micromo.com/>  
Globe Motors <http://www.globe-motors.com/home.html>

### Design Parameters for 12 Volt Brush Commutated DC Motor

Motor Length (Excluding Shaft)	2-1/4" or less
Motor Diameter	1-1/4" or less
Motor Weight	7 ounces or less
Unit Cost (Single Purchase)	\$100.00 or less

### Functional Requirements

Continuous Torque	= 2 oz-in. +/- 0.1 oz-in.
Torque Constant $K_T$	= 1.75 oz-in/Ampere or more
Back Emf Constant $K_E$	= 1.3 volts/Krpm or more
No load current	= 0.3 Ampere or less
Stall Current (Max)	= 6.0 Amperes or less
No Load Speed	= 7000 – 9000 rpm
Continuous Power Output	= 10 Watts or more

# Assessment Rubric for Lesson 3

This rubric is offered as an example only. Each instructor is encouraged to create their own assessment tools with respect to the needs and expectations of their individual students/classes.

Proficiency	Meets/Exceeds Requirement	Meets Some of the Requirement	Meets little or None of the Requirement
<b>Demonstrates</b> the ability to read key DC motor Data from manufacturer's published materials			
<b>Researches</b> locates and understands DC motor specifications provided by more than 3 other manufacturers.			
<b>Gathers, evaluates, and compares</b> DC motor data from several manufacturers in order to determine the best choice for a given set of design parameters or functional requirements.			
<b>Presents</b> clear and accurate description of Torque Constant, Back-EMF constant, Stall Torque and No Load RPM			
<b>Creates</b> a slide show and presentation that describes and compares several motors with similar specifications.			
<b>Completes</b> all class work, assignments and notebook requirements associated with this lesson			

Performance assessment.

- Portfolio (An organized chronology of individual achievement. This could be a notebook or a web page or a multimedia presentation)
- Work Sheets, Labs and design challenges.
- Examples of Spread Sheets to Solve Chain and Sprocket Related Problems
- Tests and Quizzes

## Student Response/Journal Entry/Assignments

- 1.) Photographs of working test apparatus and individual projects.
- 2.) Motor Manufacturer's Research Results and/or presentation.
- 3.) Mathematical Models.
- 4.) Tests or Quizzes.